Surface growth models with a random-walk-like nonlocality

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To understand the effects of a random-walk-like nonlocality on the dynamical scaling properties of surface growths, a stochastic growth model in which the height difference $\triangle h_{(i,i+1)}=|h_i-h_{i+1}|$ of a chosen nearest neighbor column pair $(i,i+1)$ is decreased by one unit is introduced and studied by simulations. The probability $P_{(i,i+1)}$ of choosing a column pair $(i,i+1)$ on a one-dimensional substrate is assigned as $P_{(i,i+1)}=e^{\kappa\triangle h_{(i,i+1)}}/\sum_{j=1}^{L}e^{\kappa\triangle h_{(j,j+1)}}$. On a substrate of given size L, the dynamical scaling property satisfies a normal scaling behavior as $W=L^{\alpha}f(t/L^{z})$, when κ is very small. If κ becomes moderately large, the scaling property with the dynamic exponent $z=1$ as in diffusion-limited erosion appears. If κ becomes very large, no surface roughening is found.

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I. INTRODUCTION

Recently, many dynamical scaling theories for surface growth under thermal white noise have been investigated due to both theoretical and experimental importance for the long time, large scale surface morphology [1]. The dynamical scaling hypothesis used in these studies is

$$W=L^{\alpha}f(t/L^{z}), \tag{1}$$

where W is a root-mean-square fluctuation of surface heights, α is the roughness exponent, and z is the dynamic exponent. The scaling function satisfies $f(x)\rightarrow$const. for $x\gg 1$ and $f(x)\sim x^{\beta}$ $(\beta\equiv\alpha/z)$ for $x\ll 1$ [1]. Among the scaling theories [1] many studies [2–13] have been focused on the linear growth equations

$$\frac{\partial h_{\mathbf{q}}(t)}{\partial t}=-\nu|\mathbf{q}|^{z}h_{\mathbf{q}}(t)+\eta_{\mathbf{q}}(t), \tag{2}$$

$$\frac{\partial h(\mathbf{x},t)}{\partial t}=-\nu(-\nabla^{2})^{z/2}h(\mathbf{x},t)+\eta(\mathbf{x},t). \tag{3}$$

In particular, Edwards-Wilkinson (EW) equation with $z=2$ [3] and the related discrete growth models [2,4] have been well understood now. Moreover the stochastic growth models [4–8], which are believed to follow the Mullins-Herring [9] equation with $z=4$, have now been understood to a certain maturity level. In contrast, the scaling properties for Eq. (2) with $z=1$ [10,11] have not been understood so well and only a few stochastic growth models [10–13] related to Eq. (2) with $z=1$ have been suggested and studied.

In the unit evolution process of normal surface evolution models [1], a column is randomly selected and the evolution occurs only around the chosen column. This randomness is reflected by the second term of Eq. (2) as a local white noise, where $\eta_{\mathbf{q}}(t)$ is a Fourier component of Gaussian white noise that has zero mean $\langle\eta_{\mathbf{q}}(t)\rangle=0$ and covariance $\langle\eta_{\mathbf{q}}(t)\,\eta_{\mathbf{q}'}(t)\rangle=DL^{-d}\delta_{\mathbf{q},-\mathbf{q}'}\delta(t-t')$. Therefore the unit evolution process in many models depends on the local morphology near the randomly selected column [1]. Among such models is the Ballistic erosion (BE) model [12,14,15] which is known to belong to EW universality class. In BE model, a particle from outside of a material comes straight down along a randomly selected column and it knocks out the first particle which it encounters. [See Fig. 1(a).] In BE model evolution processes are local processes, which make the model belong to EW universality class.

In contrast the models [10–13] which have been suggested to follow the linear equation (2) with $z=1$ have a nonlocal relaxation dynamics. The diffusion-limited erosion (DLE) model [10–12] and time-reversed dielectric breakdown model (TDBM) [13] have such nonlocality in selecting a site for the evolution. In DLE [10–12], a particle which starts far from the existing material undergoes a random walk until it touches a material particle on the surface. Then the two particles disappear by the reaction such as $A+B\rightarrow 0$ [12]. The incoming particle in DLE has more chance to touch some protruded part of surface than a flat part because of the nonlocal nature of random walks. [See Fig. 1(b).] The DLE model thus has random-walk-like nonlocality in selecting a site where an evolution process takes place. This nonlocal behavior makes the surface less rough and follows Eq. (2) with $z=1$.

The space-time dependence of density $\phi(\mathbf{x},t)$ of random walkers follows the diffusion equation $\partial\phi/\partial t=D_{\phi}\nabla^{2}\phi$. The probability that an incoming particle reaches a site on the

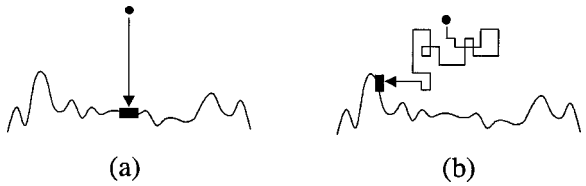

FIG. 1. Schematic diagram for comparison of the local noise (a) to the random-walk-like nonlocality (b) in erosion models. (a) The erosive process in the model with local noise occurs directly at a randomly chosen column as in ballistic erosion model. (b) The erosive process has more chance to occur at some protruded part of surface when a random walker mediates the process as in DLE model.

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surface is proportional to $|\nabla\phi|$ in the DLE model. From this theoretical basis we recently suggested TDBM [13] in which the evaporation probability of a particle on the surface is assigned to be proportional to $|\nabla\phi|$, where ϕ is the steady-state solution of the diffusion equation with a proper boundary condition. In TDBM the evaporation probability at a protruded part of surface is larger than at a flat part because $|\nabla\phi|$ becomes larger at the protruded part of surface. The dynamic scaling behavior of TDBM has been shown to be the same as that of DLE. From these models, we can understand that the random-walk-like nonlocality is essential to the scaling behavior which follows Eq. (2) with $z=1$.

However, these models [10–13] with such nonlocality to give the dynamic exponent $z=1$ are all erosive models, where the surface height is always decreasing. Furthermore the erosive process in these models is not like a common process of most growth model which changes surface height $h(\mathbf{x},t)$ directly, but is dependent on a random walker or density field gradient $|\nabla\phi|$. This indirect process makes it very difficult to understand how the dynamical scaling behavior with $z=1$ arises. Thus, our purpose in this paper is to establish a more simple growth model which has the dynamical scaling behavior with $z=1$ and the random-walk-like nonlocality as DLE and TDBM. The simple growth model for our purpose means the model in which the surface height is always increasing and $h(\mathbf{x},t)$ is directly changed.

II. MODEL

We now want to explain our growth model in detail. Our model is defined only on a one-dimensional (1D) substrate, but generalization to those on higher-dimensional substrates can be easily obtained from the definition of 1D model. As can be seen in Eqs. (2) and (3), the change of $h(x,t)$ for $z=1$ scaling behavior should depend on the magnitude of local slope, $|\nabla h(x,t)|$. A discrete version of the local slope $|\nabla h(x,t)|$ is $\Delta h_{(i,i\pm1)}=|h(i,t)-h(i\pm1,t)|$. From this observation, we can notice that the height change must depend on the height difference $\Delta h_{(i,i+1)}$ for $z=1$ scaling behavior.

The growth rule for our purpose can thus be established as what follows. Consider the surface configuration described by integer height variables $\{h(i,t)\}$, where i is an integer parameter which describes the ith column on a 1D chain. Then a column pair $(i,i+1)$ is chosen based on the probability assignment $P_{(i,i+1)}$ as

$$P_{(i,i+1)}=\frac{e^{\kappa\triangle h_{(i,i+1)}}}{\sum_{j=1}^{L} e^{\kappa\triangle h_{(j,j+1)}}}, \tag{4}$$

where L is the size of substrate. Then take the following growth process at the chosen column pair $(i,i+1)$; $h(i+1,t)\rightarrow h(i+1,t)+1$ if $h(i,t)\geqslant h(i+1,t)$, or $h(i,t)\rightarrow h(i,t)+1$ if $h(i,t)<h(i+1,t)$. This growth process should decrease the height difference $\Delta h_{(i,i+1)}$ as $\Delta h_{(i,i+1)}\rightarrow\Delta h_{(i,i+1)}-1$. Of course we use the lateral periodic condition $h(i+L,t)=h(i,t)$. Unlike local model the column pair for a growth process is not chosen randomly. The selection of the pair in the system depends on its $\Delta h_{(i,i+1)}$ through Eq. (4). Since the probability assignment has such nonlocal nature, the column pair with larger value of $\triangle h_{(i,i+1)}$ has more chance to be selected for growth than that with smaller value of $\triangle h_{(i,i+1)}$. This nonlocal nature of our model is very similar to those of DLE or TDBM in the sense that the more protruded part can be chosen for the evolution with the more chance. Thus, the global model with an appropriate range of κ is expected to have scaling behavior with $z=1$, which will be shown.

We can also consider a local version of our model in which the pair for growth is chosen randomly as in the BE model. We can establish local model as follows; first select a column l randomly. Then $P_{(l,l+1)}$ or $P_{(l-1,l)}$ which selects a nearest neighbor column pair between two column pairs $(l,l+1)$ and $(l-1,l)$ is defined as

$$P_{(l,l+1)}=\frac{e^{\kappa\triangle h_{(l,l+1)}}}{\sum_{m=\pm1} e^{\kappa\triangle h_{(l,l+m)}}}, \quad P_{(l-1,l)}=1-P_{(l,l+1)}. \tag{5}$$

Of course the other growth process at the selected column pair is the same as that of the global model. In this local model the growth process is very similar to other local models, because a column is randomly chosen and the growth process centered at the chosen column always takes place.

III. RESULTS

Now let us explain the simulation results. We perform numerical simulations, starting from a flat surface of linear size L with the lateral periodic boundary condition. We measure the surface fluctuation width W as

$$W^2(L,t)=\frac{1}{L}\sum_{i=1}^{L}\left\langle\left[h(i,t)-\frac{1}{L}\sum_{i=1}^{L}h(i,t)\right]^2\right\rangle. \tag{6}$$

All data are taken by averaging over more than 100 independent runs.

We first want to mention simulation results for the local model with assignment (5) briefly. Because the unit growth process in the local model reduces the local slope at the randomly chosen column, the local model should have the negative local current [16] as the models which belong to EW universality class [1–3]. We can thus guess that the local model belongs to EW universality class regardless of the value of κ. We confirm it by checking $\alpha=1/2$ and $\beta=1/4$ for any κ in the simulations.

Next we discuss simulation results for the global model with the probability assignment (4). The first result to discuss is the dependence of W on κ for a fixed system size L. For small κ, a normal scaling behavior as Eq. (1) is found. One example is shown in Fig. 2(a). In Fig. 2(a), we display the early-time ($t\ll L^z$) results for $\kappa=2$ and $L=256$. The growth exponent β is estimated by fitting the data to the relation $W\simeq t^\beta$ and obtained result is $\beta=0.25(1)$. The power law behavior $W(t)\simeq t^\beta$ with β close to EW value (or $\beta=1/4$) is confirmed to exist for $\kappa=2$ on substrates with sizes from L

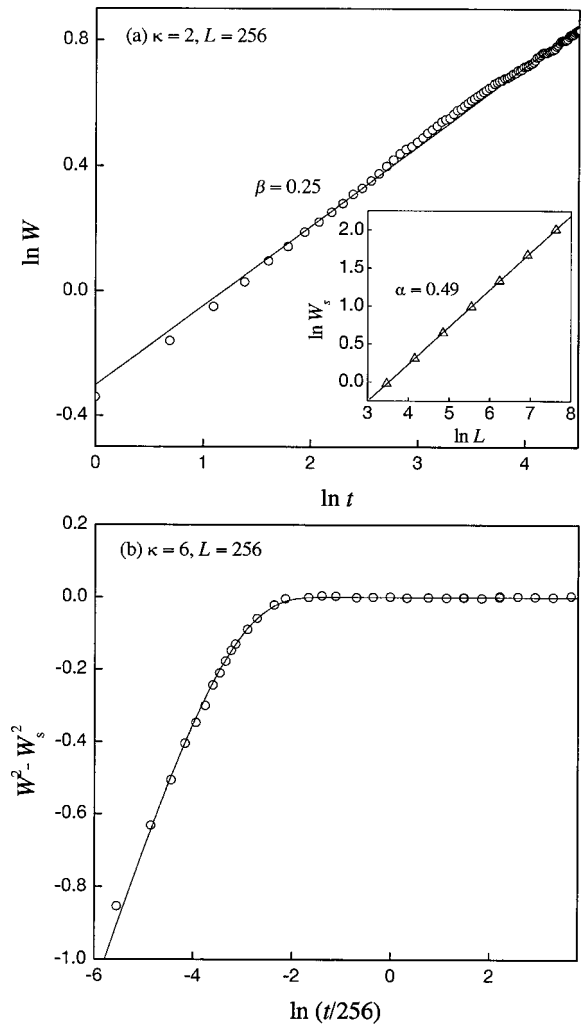

FIG. 2. (a) Plot of W against t for $\kappa=2$ and $L=256$ in the early-time regime ($t \ll L^z$). The straight line represents the relation $W \sim t^{\beta}$ with $\beta=0.25$. The inset shows the plot of W_s for $\kappa=2$ against L in the saturation regime ($t \gg L^z$). The straight line denotes the relation $W_s \sim L^{\alpha}$ with $\alpha=0.49$. Used system sizes are $L=2^5, 2^6, 2^7, 2^8, 2^9, 2^{10}$, and 2^{11}. (b) Plot of $W^2-W_s^2$ against $\ln(t/256)$ for $\kappa=6$ and $L=256$. The solid curve is the fitting of data to the relation (7) or (8), which shows that W^2 follows nicely the linear equation (2) with $z=1$.

$=2^5$ up to $L=2^{11}$. Furthermore we study the behavior of saturated W, W_s [$=W(t \gg L^z)$], by using the substrates with sizes $L=2^5, \ldots, 2^{11}$. We find that W_s for $\kappa=2$ also satisfies usual scaling behavior $W_s \simeq L^{\alpha}$ with $\alpha=0.49(1)$, which is also very close to EW value ($\alpha=1/2$) as shown in the inset of Fig. 2(a). It is also found that this kind of normal EW scaling behavior exists for $0<\kappa<\kappa_p(L)$ in the system with a fixed size L.

For $\kappa>\kappa_p(L)$ we find two different regimes. In the moderate range of κ or in $\kappa_p(L)<\kappa<\kappa_r(L)$ the scaling behavior of W is found to follow Eq. (2) with $z=1$ as DLE or TDBM. For the specifics see Fig. 2(b), where the scaling behavior of W for $\kappa=6$ ($>\kappa_p$) on the substrate with $L=256$ is displayed. $W(t)$ in Fig. 2(b) is shown to nicely follow the solution of Eq. (2) with $z=1$ [10,11],

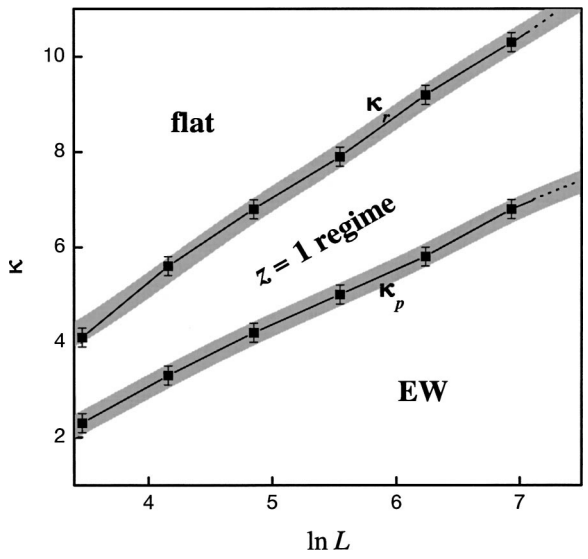

FIG. 3. Dependence of κ_p and κ_r on the system size L. κ_p and κ_r are estimated in the systems with the sizes $L=2^5, 2^6, \ldots, 2^{10}$. The solid lines between data points and the dashed line above $L=2^{10}$ are from simple interpolations and extrapolations. The shaded area can be regarded as the crossover region from one regime to another regime. This figure can also be the estimated phase diagram for EW, $z=1$ and flat-surface regimes.

$$W^2=\frac{D}{2\pi\nu}\left\{\ln\left(\frac{L}{a}\right)+\ln\left[1-\exp\left(\frac{-4\pi\nu t}{L}\right)\right]\right\} \tag{7}$$

or

$$W^2=W_s^2+\frac{D}{2\pi\nu}\left\{\ln\left[1-\exp\left(\frac{-4\pi\nu t}{L}\right)\right]\right\}. \tag{8}$$

by plotting $(W^2-W_s^2)$ against $\ln(t/L)$. Here D comes from the covariance $\langle \eta_{\mathbf{q}}(t)\eta_{\mathbf{q}'}(t)\rangle = DL^{-d}\delta_{\mathbf{q},-\mathbf{q}'}\delta(t-t')$ of the noise $\eta_{\mathbf{q}}(t)$, ν is one of the coefficients in Eq. (2), and a is the short range cutoff distance [10,11]. a in our simulation should be the unit lattice spacing and thus $a=1$. Of course $\kappa_r(L)$ also depends on the system size L. If κ becomes large or $\kappa>\kappa_r(L)$, no roughening regime or the flat-surface regime appears.

To see the dependence of κ_p and κ_r on the system size L, we estimate κ_p and κ_r for $L=2^5, \ldots, 2^{10}$. The results are displayed in Fig. 3. The solid lines between the data and the dashed line above $L=2^{10}$ in Fig. 3 are from simple interpolations and extrapolations. Figure 3 can be also regarded as the phase diagram for EW, $z=1$ and flat-surface regimes in κ-L parameter space. Both $\kappa_r(L)$ and $\kappa_p(L)$ seem to be monotonically increasing functions of L. In the thermodynamic limit ($L\to\infty$) both κ_p and κ_r go to infinity. The model then always crossovers to EW universality class in the limit $L\to\infty$ for a given κ. To get the $z=1$ regime in the $L\to\infty$, one must take the limit $\kappa\to\infty$ carefully, so that (L,κ) should be in the $z=1$ regime while taking both the limits. It is found that the crossover behavior between the regimes is somewhat broad. (See the shaded regions in Fig. 3.) Because we do not have a physically sound theory for the finite size effects, we

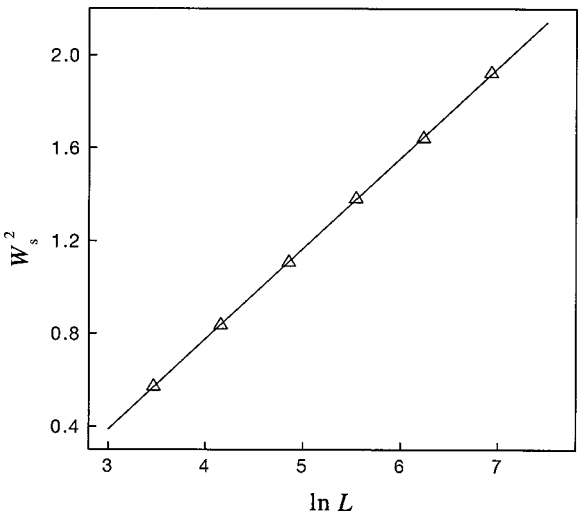

FIG. 4. Plots of W_s^2 against ln$L$ for a set of (L, κ) combinations; (2^5, 3.3), (2^6, 4.0), (2^7, 4.9), (2^8, 5.9),(2^9, 7.0), and (2^{10}, 8.1). The data nicely follow Eq. (9). The slope of the fitted line gives $D/2\pi\nu=0.390(2)$.

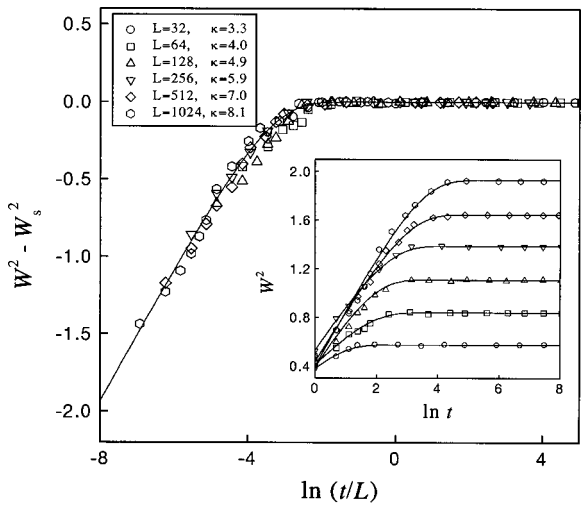

FIG. 5. Plots of $W^2-W_s^2$ against ln(t/L) for the same set of (L, κ) combinations in Fig. 4. The inset shows W^2 for each combination nicely follows Eq. (7) or (8) using $D/2\pi\nu=0.390(2)$ which is obtained from Fig. 4. The main plot shows that all the data for the set collapse well to one curve based on the scaling relation (7) with $D/2\pi\nu=0.390(2)$.

cannot tell that such a crossover comes from the intrinsic character of the model or the finite size effects. Further study to characterize the crossover is left for future research.

The existence of such three regimes or the existence of κ_p and κ_r can be understood from Eq. (4). Even though $\Delta h_{(i,i+1)}$ is large, the probability $P_{(i,i+1)}$ to choose the pair $(i,i+1)$ for small κ ($\kappa<\kappa_p$) cannot become large enough to have the random-walk-like nonlocality. This fact means that the protruded part cannot be taken with the same probability as that in DLE or TDBM for small κ. Therefore, in the range $\kappa<\kappa_p$ a column pair for growth is selected almost randomly and the growth process is nearly the same as that in the local model. This is the physical reason why the models for $\kappa<\kappa_p$ belong to EW universality class with $\alpha=1/2$ and $z=2$. For the moderately large κ or in the range $\kappa_p<\kappa<\kappa_r$, $P_{(i,i+1)}$ for the large $\Delta h_{(i,i+1)}$ is enhanced enough to show the random-walk-like nonlocality as in DLE or TDBM, so that the scaling behavior with $z=1$ appears. This behavior comes from the term $e^{\kappa\Delta h_{(i,i+1)}}$ in Eq. (4). In other words the random-walk-like behavior can be understood from the stochastic behaviors of the probability assignment (4) in the range $\kappa_p<\kappa<\kappa_r$. For $\kappa>\kappa_r$, $P_{(i,i+1)}$ for $\Delta h_{(i,i+1)}\neq 0$ becomes very large and we can expect the layer-by-layer growth, where the surface becomes unroughened.

We now want to characterize the dynamic scaling behavior of $z=1$ regime more specifically. $W(L,t)$ in $z=1$ regime of Fig. 3 satisfies Eqs. (7) and (8). However the constant $D/2\pi\nu$ varies as κ varies within the $z=1$ regime in Fig. 3. Furthermore to obtain one common value for the coefficient $D/2\pi\nu$ in different system sizes L, κ should be carefully tuned in the κ-L parameter space. To see this effect, the dependence of W_s^2 of Eqs. (7) and (8) on L

$$W_s^2=\frac{D}{2\pi\nu}\ln(L/a)=\frac{D}{2\pi\nu}\ln L, \tag{9}$$

must be taken into consideration. In Eq. (9) we set $a=1$, which is the short range cutoff distance. In Fig. 4 we display W_s's for a certain set of combinations of L and κ values, which satisfies Eq. (9) with a fixed value of $D/2\pi\nu$ well. The set consists of ($L=2^5$, $\kappa=3.3$), (2^6, 4.0), (2^7, 4.9), (2^8, 5.9), (2^9, 7.0), and (2^{10}, 8.1). From the fitting of Eq. (9) to W_s in Fig. 4, we get $D/2\pi\nu=0.390(2)$ for the set of (L, κ) combinations. In Fig. 5, we plot $W^2-W_s^2$ against ln(t/L) for the same set in Fig. 4. In Fig. 5 the solid curve from the scaling relation based on Eq. (7) with $D/2\pi\nu=0.390(2)$ is shown to explain the dynamical data of $W(t)$ for the set of (L, κ) combinations very well. This kind of behavior is confirmed for other well-tuned sets of (L, κ) combinations. The results in Figs. 4 and 5 show that κ should be varied properly for various L to obtain $z=1$ scaling behavior with a fixed $D/2\pi\nu$. In DLE and TDBM, W shows exactly the same scaling behavior as Fig. 5 without parameter tunings. The scaling behavior in DLE and TDBM comes from the intrinsic random-walk-like nonlocality, which comes from the steady-state diffusion equation, i.e., Laplace equation [10,11,13]. The scaling behavior of DLE and TDBM thus comes from the Laplace equation with proper boundary conditions. From the view of our model we can consider that the nonlocality from the Laplace equation naturally and spontaneously selects the parameters for the $z=1$ scaling behavior with the naturally fixed $D/2\pi\nu$.

IV. SUMMARY AND DISCUSSIONS

In summary, we studied the surface fluctuation W for growth models with the probability assignment (4). For $\kappa<\kappa_p$, W satisfies the normal scaling behavior (1) with the EW exponents $\alpha=1/2$ and $z=2$. For $\kappa_p<\kappa<\kappa_r$, W follows Eq. (7) nicely, which is the solution of the linear equation (2) with $z=1$. These results mean that the models with $\kappa_p<\kappa$

$<\kappa_r$ have the same random-walk-like nonlocality as DLE and TDBM. For $\kappa>\kappa_r$ no surface roughening is found. κ_r and κ_p depend on L. Furthermore κ should be varied properly for different L in $\kappa_p<\kappa<\kappa_r$ to obtain $z=1$ scaling behavior with a fixed $D/2\pi\nu$. In conclusion the effect of the random-walk-like nonlocality on dynamical structure can be understood directly from Eq. (4). The range of κ in which the scaling behavior with $z=1$ or the random-walk-like nonlocality occurs is determined by the factor $e^{\kappa\triangle h_{(i,i+1)}}$ in Eq. (4), because $e^{\kappa\triangle h_{(i,i+1)}}$ for a pair with large $\triangle h_{(i,i+1)}$ cannot become large enough to show the random-walk-like nonlocality when κ is small or $\kappa<\kappa_p$.

Final discussions are on three points. One is on the generalized version of the local model with Eq. (5). In the local model a column pair for growth is selected only between the two pairs centered at the randomly chosen column l. Instead we can consider n $(<L)$ pairs with the randomly chosen column l at the center. Then the probability assignment should be modified as $P_{(i,i+1)}=e^{\kappa\triangle h_{(i,i+1)}}/\sum_{m=-n/2}^{n/2-1}e^{\kappa\triangle h_{(l+m,l+m+1)}}$. This modified version of the model is quite similar to the erosion model, in which the incoming particle undergoes a biased random walk to the material [12]. If the particle is biased to the material, then the lateral distance to sweep for the particle is limited before touching surface and thus the erosive process only occurs on the columns within the limited lateral distance centering the column above which the particle starts. If $n/L\to 0$, the modified model with the n pairs for selection is confirmed to belong to EW universality class as the erosion model with the biased random walks [12].

The second one is the functional dependence of the probability assignment $P_{(i,i+1)}$. In Eq. (4) $P_{(i,i+1)}$ is set as $P_{(i,i+1)}\propto e^{\kappa\Delta h_{(i,i+1)}}$. One of the natural variants is $P_{(i,i+1)}\propto\kappa\Delta h_{(i,i+1)}+\delta$, where the constant δ is added in order to prevent $P_{(i,i+1)}$ from being everywhere zero for the initial flat surface or the surface with all $h(i)=0$. We confirmed that the variant model shows $z=1$ scaling behavior if δ becomes moderately small. In $\delta\to 0$ limit, we also confirmed the layer-by-layer growth.

Final discussion is on another possible way to make the random-walk-like nonlocality in the surface growth model. In the model considered so far the selection probability of a column pair is proportional to $e^{\kappa\Delta h_{(i,i+1)}}$ and the growth always occurs at the chosen pair. Another possible model is as what follows. First select a pair randomly. Second the growth at the randomly chosen pair is accepted with the rate proportional to $e^{\kappa\Delta h_{(i,i+1)}}$. This modified version of the model is physically the same as the model considered in this paper. We realized the second step in the modified version of model by assigning acceptance probability P_a of the growth at the randomly chosen pair as $P_a=e^{\kappa\Delta h_{(i,i+1)}}/e^{\kappa\Delta h_{max}}$, where Δh_{max} is the maximum of $\Delta h_{(i,i+1)}$ in the given surface configuration. We also confirmed the $z=1$ regime in a certain range of κ in the modified model. This modified model also suggests another physical way to understand the random-walk-like locality.

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